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# Performance Evaluation of Carrier Interferometry Implementations of MC-CDMA Over a Wideband Channel Suffering Phase Noise

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**Abstract** – In this paper the performance of Carrier Interferometry (CI) and Pseudo-Orthogonal Carrier Interferometry (PO-CI) [1] implementations of Multi-Carrier Code Division Multiple Access (MC-CDMA) are investigated. Comparisons are made with the performance of traditional MC-CDMA (incorporating Hadamard-Walsh spreading codes) [2],[3] and Coded Orthogonal Frequency Division Multiplexing (COFDM) [4]. These systems are simulated over a 20MHz bandwidth, wideband, wide-sense stationary, statistically uncorrelated Rayleigh fading channel suffering additive white Gaussian noise (AWGN) and phase noise. The two channel models employed are consistent with the Model B ‘Indoor and Office’ and Model B ‘Outdoor to Indoor and Pedestrian’ test environment specifications proposed by ETSI for UMTS Terrestrial Radio Access (UTRA) [5].

The BPSK CI system was found to outperform both of the equivalent COFDM and MC-CDMA systems, providing 1dB and 1.5dB performance gains respectively at a bit error rate (BER) of  $10^{-3}$ . The BPSK PO-CI system, shown to offer twice the capacity of the equivalent MC-CDMA, COFDM and CI systems, was found to offer the same performance as MC-CDMA.

For higher order modulation schemes, the CI system’s performance was found to equal that of the equivalent MC-CDMA system, whilst the performance of PO-CI was found to be very poor.

In the presence of phase noise, the CI system was found to outperform the MC-CDMA and COFDM systems, suffering a far lower performance loss for a given level of phase noise and providing a softer fail as this level was increased. This soft fail characteristic was also observed in the PO-CI system.

**Keywords** - Carrier interferometry; phase noise; multi-carrier code division multiple access (MC-CDMA); frequency domain spreading.

## I. INTRODUCTION

The OFDM transmission system reduces the effects of multipath delay spread inherent in wideband transmissions through symbol duration extension by the serial to parallel conversion of the input symbol stream into blocks of  $N$  symbols (where, for a system with modulation index  $q$ , each symbol represents  $q$  data bits). These symbols are then simultaneously modulated onto  $N$  corresponding sub-carriers. This allows the overall transmission rate and spectrum use to remain unchanged, whilst reducing the bandwidth of any given

sub-carrier to  $(1/N)^{\text{th}}$  of the total system bandwidth. The number of sub-carriers and hence the sub-carrier bandwidth may then be selected to produce narrowband sub-carrier transmissions, each experiencing flat fading upon transmission over the frequency selective channel. However, due to the inherently wideband nature of the overall transmission system, it is likely that one or more of the constituent sub-carriers will be in a deep fade at any given time. Data symbols transmitted on such carriers will be corrupted, producing symbol errors upon reception. This problem is overcome in COFDM, by the use of channel coding and interleaving, prior to symbol mapping. This introduces redundancy and frequency diversity at the expense of effective transmission rate.

Multi-Carrier Code Division Multiple Access (MC-CDMA), the frequency domain analogue of DS-SS, offers a way to harness the frequency diversity gains inherent in COFDM based systems, such as those based on the IEEE802.11a/g/n, IEEE802.15.3a MBOA and DVB-RCT standards, whilst providing the capacity gains observed in and beyond the current 3<sup>rd</sup> Generation (3G) CDMA cellular systems [6],[7],[8]. Here, each user is assigned a Hadamard-Walsh spreading code, which is applied to their data symbols in the frequency domain, spreading each symbol across multiple sub-carriers and thus introducing frequency diversity. The orthogonality of the codes allows the transmitted symbols to be separated upon reception.

A recently proposed technique, known as Carrier Interferometry, offers a powerful alternative to traditional MC-CDMA. In this system the antipodal Hadamard-Walsh spreading code chips, inherent in the MC-CDMA system, are replaced with complex spreading code chips. The relaxation of this antipodal chip constraint provides an extra degree of freedom within the CI spreading scheme, allowing the generation of a second spreading code set, ‘pseudo-orthogonal’ to the first. The parallel application of these two code sets produces a system with twice the capacity of a traditional MC-CDMA system, whilst introducing a small and calculable level of interference due to their pseudo-orthogonal nature.

The performance of COFDM based systems can be severely reduced by the presence of random phase noise in oscillators, causing constellation rotation (common phase error,

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(CPE)) and intercarrier interference (ICI) [9],[10]. These effects become a serious problem in systems incorporating high data rates at high carrier frequencies.

Section II provides an overview of the spreading code techniques and the resulting OFDM symbol waveforms; Section III presents the transmitter and receiver models and outlines the spreading scheme dependant system parameters; Section IV discusses the channel and phase noise models employed in system simulations, whilst Sections V and VI present the performance results and conclusions respectively.

## II. SPREADING MODELS

### A. OFDM and COFDM

Due to the simultaneous transmission of  $N$  data symbols, the OFDM system has an OFDM symbol period  $T_u$ , which is a factor of  $N$  larger than the input symbol stream period  $T$ . Thus, the normalised delay spread for such a system is reduced by a factor of  $N$ , producing a system which is extremely tolerant to multipath delay spread. The transmitted signal for an  $N$  carrier COFDM symbol, consisting of  $qN$  coded data bits, mapped to  $N$  symbols,  $a[k]$ , for  $k=[0 \dots N-1]$  is given by (1).

$$s(t) = \sum_{m=0}^N c_k[m] a[k] e^{2\pi j(f_c + m\Delta f)t} \quad (1)$$

where  $c_k[m] = \delta_{k,m}$ , modulating a single symbol,  $a[k]$ , onto each sub-carrier.  $f_c$  represents the fundamental carrier frequency.  $\Delta f = 1/T_u$  is the intercarrier spacing required to ensure orthogonality among sub-carriers, where  $T_u = N/B$  is the active OFDM symbol period for a system with total bandwidth  $B$ .

### B. MC-CDMA

MC-CDMA, the frequency domain analogue of DS-SS, spreads each symbol of a transmission across multiple sub-carriers using orthogonal Hadamard-Walsh spreading codes. The orthogonality of these codes allows the transmitted symbols to be separated upon reception. Each user,  $l=[0..L-1]$ , transmitting data symbols  $k=[0..K-1]$ , is assigned a unique spreading code vector of length  $SF$ , such that  $K=N/SF$  and  $LK \leq N$ . For each user,  $K$  vectors are formed by the multiplication of the user's spreading code by their  $K$  data symbols. The resulting vectors are concatenated to form a vector of length  $N$ , where each element represents the contribution of the user's data to a corresponding carrier modulation coefficient. The spread data vectors are then summed over the  $L$  users to form an overall vector of  $N$  modulation coefficients, representing the cumulative effect of all  $L$  users' modulation contributions. This final multi-user modulation vector is applied to the  $N$  carriers over an OFDM symbol period of length  $T_u$ .

The transmitted signal for an  $N$  carrier MC-CDMA symbol, supporting  $L$  users, with  $K$  data symbols per user is given by (2).

$$s(t) = \frac{1}{\sqrt{SF}} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \sum_{p=0}^{SF-1} c_l[p] a_l[k] e^{2\pi j(f_c + (p+kSF)\Delta f)t} \quad (2)$$

where  $c_l[p] \in \{-1, 1\}$  represents the  $p^{\text{th}}$  chip of the  $l^{\text{th}}$  users' spreading code,  $SF$  represents the length of the spreading codes and  $SF^{-1/2}$  scales the waveform amplitude in order to ensure a bit energy of unity.  $a_l[k]$  represents the  $k^{\text{th}}$  data symbol of the  $l^{\text{th}}$  user and  $LK \leq N$ .

### C. Carrier Interferometry

Carrier interferometry represents a more generalised orthogonal spreading technique in which the spreading code chips are not constrained to antipodal values. In the CI system, the chips from a given code form a set of phase codes which increase linearly with frequency (i.e. carrier index). For  $N$  carriers the fundamental rate of this phase increase is set at  $2\pi/N$  per carrier increment, such that the chips are spaced equally within the phase domain. Codes produced by increasing the phase at integer multiples of this rate are orthogonal and are equivalent to applying a temporal displacement to the fundamental waveform of  $nT_u/N$  where  $n \in \mathbb{Z}$ . Using this technique, a fundamental set of  $N$  unique orthogonal codes can be produced, given by (3).

$$c_l[m] = e^{2\pi j m l / N} \quad (3)$$

where  $m=[0..N-1]$  represents the carrier (and chip) index and  $l=[0..N-1]$  produces the  $N$  unique codes.

The transmitted signal for an  $N$  carrier CI symbol, supporting  $L$  users with  $K$  data symbols per user is given by (4)

$$s(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} a_l[k] e^{j2\pi \left( (f_c + m\Delta f)t + \frac{m(Ll+k)}{N} \right)} \quad (4)$$

where  $a_l[k]$  represents the  $k^{\text{th}}$  data symbol of the  $l^{\text{th}}$  user,  $\Delta f = 1/T_u$  represents the intercarrier spacing required to ensure orthogonality between sub-carriers, the  $N^{1/2}$  term is a scaling factor used to produce a bit energy of unity and  $LK \leq N$ .

### D. Pseudo-Orthogonal Carrier Interferometry

Whilst codes created using non-integer multiples of the fundamental rate are not orthogonal to members of the fundamental set, the mutual interference between two such codes is small when their rates vary by an odd multiple of half the fundamental rate, as given by (5) [11].

$$c_n[m] = c_l[m] \cdot e^{\frac{2\pi j m}{N} \left( n + \frac{1}{2} \right)} \quad (5)$$

where  $c_l[m]$  is any member of the fundamental set and  $n \in \mathbb{Z}$

This is equivalent to a temporal displacement of the fundamental waveform by  $(n+1/2)T_u/N$ . The mutual interference between two such signals is proportional to  $N^{-1/2}$  and therefore, for large  $N$ , a second set of  $N$  codes may be ‘pseudo-orthogonally’ overlaid between the members of the fundamental set. The simultaneous use of both fundamental and secondary code sets allows the transmission of up to two data symbols per carrier, doubling the capacity of the carrier interferometry system without bandwidth expansion. The transmitted signal for an  $N$  carrier PO-CI symbol, supporting  $L$  users with  $K$  data symbols per user is given by (6).

$$s(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} a_l[k] e^{j2\pi \left( (f_c + m\Delta f)t + m \left( \frac{Ll+k}{2N} \right) \right)} \quad (6)$$

where  $LK \leq 2N$ .

### III. TRANSMITTER AND RECEIVER STRUCTURES

The transmitter and receiver models used are shown in Fig. 1. The transmitter incorporates channel coding, symbol mapping and block interleaving. A guard interval of length  $N_{gi}$  is also implemented as a cyclic prefix to the OFDM symbol in order to combat the InterSymbol Interference (ISI) inducing effects of multipath delay spread. The system is characterised by  $N$  modulated sub-carriers, spread equally over bandwidth  $B$ , with sampling period  $1/B$ .

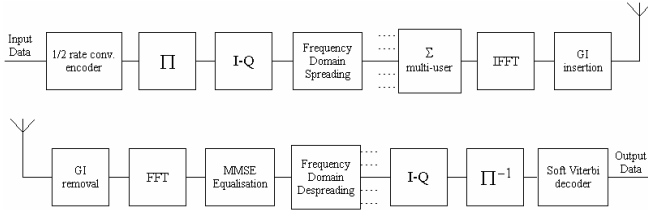


Figure 1. Transmitter and receiver structures

Hadamard-Walsh codes of length 32 are used in the MC-CDMA system, allowing up to  $SF=32$  unique, orthogonal spreading codes (and hence users). In the case of the COFDM system, the spreading code chips take the form of the trivial delta function  $c_l[p]=\delta_{l,p}$ , as this system is equivalent to an MC-CDMA system with a spreading factor of 1. The spreading codes used in the CI system are defined in (7).

$$c_l[p] = e^{2\pi j \frac{p(Ll+k)}{N}} \quad (7)$$

where  $p=[0..N-1]$  represents the chip index and  $l=[0..N-1]$  produces the  $N$  unique codes. Similarly, the spreading codes used in the PO-CI system are defined in (8).

$$c_l[p] = e^{j\pi \frac{p(Ll+k)}{N}} \quad (8)$$

Here  $l=[0..2N-1]$  produces the  $2N$  unique codes, with even and odd  $l$  values producing the (pseudo-orthogonal) fundamental and secondary code sets respectively and constituent members of a given set being orthogonal to all others within that set. In both CI and PO-CI systems,  $SF=N$ .

The generalised transmitted symbol  $x(n)$  is therefore given by (9).

$$x(n) = \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \sum_{p=0}^{SF-1} \frac{c_l[p] a_l[k]}{\sqrt{SF}} e^{2\pi j (f_c + (p+kSF)\Delta f) \left[ (n-N_{gi}) \bmod N \right] T} \quad (9)$$

for integer  $n=[0..N+N_{gi}-1]$  to produce the cyclic prefix guard interval. Thus the total symbol duration is  $(N+N_{gi})T$ , where  $T=1/B$  represents the sampling period and  $c_l[p]$  represents the  $p^{\text{th}}$  chip of the  $l^{\text{th}}$  user's spreading code for the spreading scheme employed.

Minimum Mean Square Error Combining (MMSEC) has been shown to outperform the sub-optimal MRC and EGC techniques in multi-user systems by jointly minimising inter-bit interference and noise, whilst exploiting frequency diversity [12]. This technique is therefore employed to provide frequency domain equalisation prior to the application of despreading codes.

The simulation parameters common to all four systems are given in Table I.

TABLE I. SIMULATION PARAMETERS

Parameter	Value
Bandwidth	20.48MHz
Total No. of Sub-carriers	64
Sub-carrier spacing $\Delta f$	320kHz
Sampling Period, $T$	48.8ns
Channel Coding	$1/2$ rate conv. $K=7$ , {133,171} <sub>octal</sub>
Useful Symbol Duration $T_u$	3.1 $\mu$ s
Guard Interval Duration $T_{gi}$	3.9 $\mu$ s
Total Symbol Duration $T_{symbol}$	7.0 $\mu$ s

### IV. CHANNEL MODEL

A tapped delay line model was produced for the wideband channel, assuming wide sense stationary uncorrelated scattering (WSSUS) and additive white Gaussian noise (AWGN). Thus, for each transmitted packet a set of tap weights is generated comprising zero mean Rayleigh distributed random variables, the variance's of which are dictated by the mean power delay profile of the channel. This system therefore simulates the arrival of multipaths following a known mean power delay profile and independent and identically distributed (iid) phases, following a uniform distribution over the range  $[0..2\pi]$ . Thus the amplitude at each tap for each sub-carrier is an independent and identically distributed random variable. Two channel models were implemented, with power delay profiles consistent with the Model B ‘Indoor and Office’ (rms delay spread,  $\tau_{\text{RMS}}=100\text{ns}$ ;

excess delay,  $\tau_{\text{Max}}=700\text{ns}$ ) and Model B ‘Outdoor to Indoor and Pedestrian’ ( $\tau_{\text{RMS}}=750\text{ns}$ ,  $\tau_{\text{Max}}=3.7\mu\text{s}$ ) test environment specifications proposed by ETSI for the UMTS Terrestrial Radio Access (UTRA) study [5]. Phase noise is modelled as a discrete successive Gaussian random walk or Wiener-Levy process with zero mean and variance  $2\pi\beta n$  [13]. Thus at sample period  $n$ , each constituent carrier suffers a phase offset, relative to the local oscillator, of  $\phi[n]$ , given by (10).

$$\phi[n] = \sum_{q=0}^n \theta_q \quad (10)$$

where  $\theta_q$  is a zero-mean Gaussian variable with variance  $2\pi\beta$  and  $\beta$  represents the two-sided 3-dB bandwidth of the spectrum [14].

## V. PERFORMANCE RESULTS

Performance results for the four systems under maximal occupancy for varying  $E_b/N_0$  are given in Fig. 2 for the UTRA compliant ‘Indoor and Office’ test environment channel model. This corresponds to the case in which the four systems concurrently support the maximum number of users permitted by their respective spreading schemes. Both BPSK and QPSK modulation schemes were simulated, without phase noise, though, for the sake of clarity, QPSK results are only plotted for the CI and PO-CI systems. In both the MC-CDMA and COFDM systems the QPSK performance was identical to that of the respective BPSK modulation schemes.

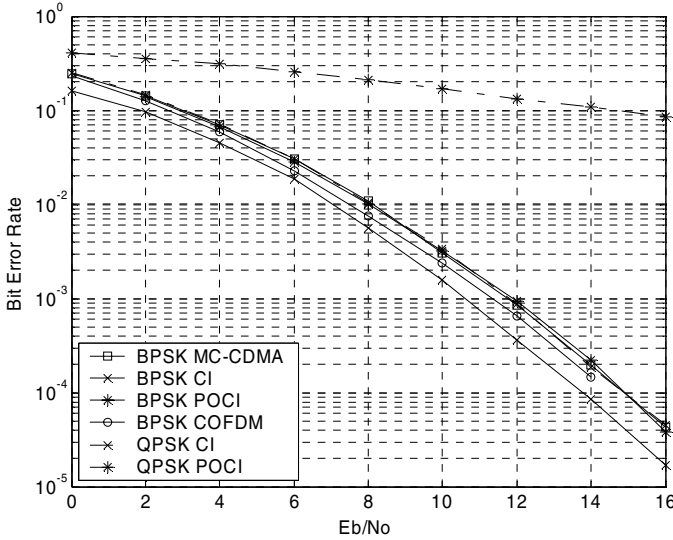


Figure 2.  $E_b/N_0$  performance results for the ‘Indoor and Office’ test environment

Performance results for the four systems under maximal occupancy for varying levels of phase noise are given in Figs. 3 and 4 for two UTRA compliant channel models. In both cases  $E_b/N_0$  was constant throughout at 10dB and a BPSK modulation scheme was employed in all four systems. Phase noise is displayed in terms of  $\beta N$ , where, as mentioned previously,  $\beta$  represents the two-sided 3-dB bandwidth of the

spectrum and  $N$  represents the size of the FFT within the receiver.

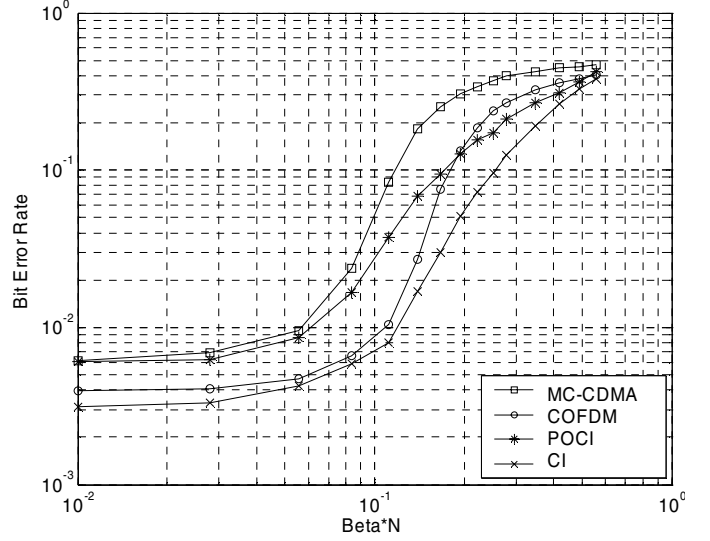


Figure 3. Phase noise performance results for the ‘Indoor and Office’ test environment

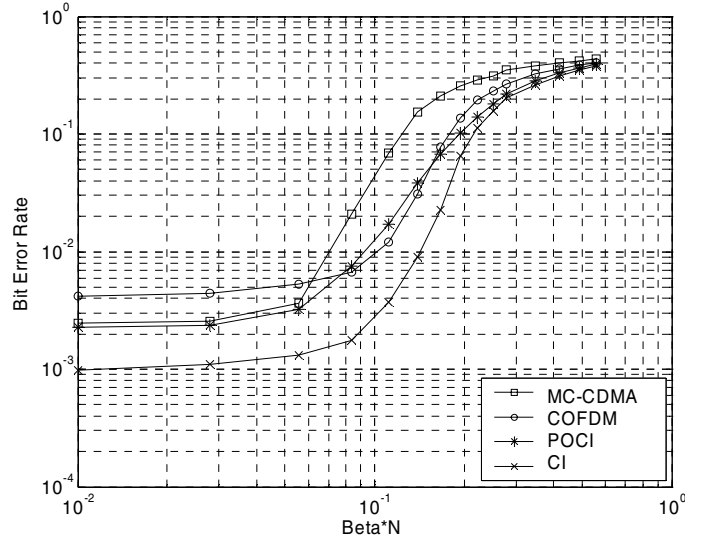


Figure 4. Phase noise performance results for the ‘Outdoor to Indoor and Pedestrian’ test environment

As can be seen from the plot of Fig. 2, at a BER of  $10^{-3}$ , the BPSK CI system outperforms the BPSK MC-CDMA and COFDM systems by 1.5dB and 1dB respectively. This is due to the nature of the CI spreading scheme, which inherently spreads the input symbol stream across both the I and Q domains. This creates a system which is analogous to a QPSK MC-CDMA system operating at half of its maximum capacity. The performance benefit observed with the application of the CI spreading scheme is due to the reduction in the interference between different user’s data symbols, caused by this extra level of separation. This performance benefit is lost with the application of QPSK CI, as both the I and Q domains are already inherently populated within the BPSK CI system.

QPSK CI therefore provides the same performance as QPSK MC-CDMA, as, in both systems, the I and Q domains are fully utilised under maximal occupancy conditions. BPSK PO-CI is analogous to the application of two CI systems across opposing I and Q domains and is therefore equivalent to, and provides the same performance as, QPSK CI. QPSK PO-CI therefore represents the superposition of two fully populated (two dimensional) I-Q domain transmissions (equivalent to the application of two QPSK CI systems applied across opposing I and Q domains). This overpopulation of the I-Q domain destroys the orthogonality between symbols, causing severe interference between the users' data and hence a highly degraded system performance.

In both the 'Indoor and Office' and 'Outdoor to Indoor and Pedestrian' test environments, the performance of the CI system is greater than that of the other three systems. Phase noise affects the system by degrading the orthogonality between data symbols within the overall CI symbol, producing interference. However, due to the inherently increased separation of the data within the CI symbol, produced by spreading the data across the entire I-Q domain, this effect is reduced relative to that observed in the equivalent MC-CDMA or COFDM systems. The CI system also incorporates phase spreading code chips whose magnitude is proportional to both user- and carrier-index. This produces a system with a large phase spacing between the spreading sequence elements of any two data symbols on the same carrier and similarly between the spreading sequence elements applied to any two carriers for a given data symbol. Any phase noise occurring within the CI system therefore represents a lower fractional error than in the equivalent MC-CDMA system and hence causes a relatively reduced effect upon despreading. The CI system is therefore more robust in the presence of phase noise, suffering a far lower performance loss for a given level of phase noise and providing a softer fail as it increases.

Though the application of the PO-CI system removes the performance benefit produced by CI's reduced interference characteristics, it still demonstrates a softer fail than both the MC-CDMA and COFDM systems, due to the large phase offsets incorporated within its spreading code chips. Thus, the PO-CI system outperforms MC-CDMA in both environments at all finite levels of phase noise.

## VI. CONCLUSIONS

Traditional, carrier interferometry and pseudo-orthogonal carrier interferometry implementations of MC-CDMA systems were simulated over two UTRA compliant wideband channels in the presence of AWGN and phase noise. The traditional MC-CDMA system was simulated with spreading factors of 32 and 1 (the latter being equivalent to COFDM).

The CI system was shown to be a powerful and versatile spreading technique, which outperforms both COFDM and

MC-CDMA while utilising BPSK modulation, and offers an equal performance to that of MC-CDMA upon the application of higher order modulation schemes. The CI system also offers greater resilience to phase noise and a soft fail characteristic.

The BPSK PO-CI system offers twice the capacity of the equivalent MC-CDMA, COFDM and CI systems and is effectively a spreading code implementation of QPSK CI. Also, further to investigations of PO-CI presented previously [1,11], it was shown that higher order modulation schemes in combination with PO-CI perform extremely poorly. Due to this and the relatively high implementational complexity of the PO-CI system, a standard QPSK CI implementation would probably be favored over a BPSK PO-CI implementation. This would offer the same performance, with the added flexibility of its permitted higher order modulation schemes.

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